Constraints on possible dynamics of QCD by symmetry and anomaly

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Yukawa Institute for Theoretical Physics (YITP), Kyoto

May 1, 2020 @ YITP

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Refs: 1807.07666, 1805.11423, 1710.08923 (1705.01949, 1711.10487, 1803.02430)

Quantum ChromoDynamics

QCD: fundamental theory of quarks and gluons

$$S = \underbrace{\frac{1}{g^2}\mathrm{tr}(F \wedge \star F)}_{\text{gluon kinetic term}} + \underbrace{\overline{\psi}\gamma_\nu D_\nu \psi + m\overline{\psi}\psi}_{\text{quark kinetic/mass terms}}$$

This is very strongly coupled theory.

Indeed, quarks/gluons never appear at low energies.

Chiral symmetry of QCD

When quarks are massless ($m = 0$), N_c-color N_f-flavor QCD enjoys chiral symmetry,

$$G = \frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_V}{\mathbb{Z}_{N_f} \times \mathbb{Z}_{N_c}}.$$

Chiral (or axial) transformation looks like

$$\psi \to \exp(\mathrm{i}\alpha T\gamma_5)\psi.$$

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Why is this important?

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Why is this important? $\Rightarrow$ Parity pairing!

$$\underbrace{\overline{\psi} T \psi}_{P\text{-even}} \xrightarrow{\text{chiral trans.}} \underbrace{\overline{\psi} T \gamma_5 \psi}_{P-\text{odd}}$$

Spontaneous chiral symmetry breaking

OK, then do we really have parity pairing?

	$I^G(J^{\text{Parity}})$	M (MeV)
$\pi^{0,\pm}$	$1^-(0^-)$	140
a_0	$1^-(0^+)$	980
f_0	$0^+(0^+)$	$\sim$500
η'	$0^+(0^-)$	958

So, our world doesn't respect parity pairing.

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So, our world doesn't respect parity pairing.

This is because of spontaneous chiral symmetry breaking,

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_F)_V.$$

Pion ($\pi^{0,\pm}$) is the Nambu-Goldstone boson ($N_f = 2$).

Let's ask the following question from curiosity.

Question

Why does QCD vacuum spontaneously break chiral symmetry?

Does dynamics choose it out of many possibility?
Or, are there any other reasons / constraints forcing it?

I know this is somewhat vague, so you can either say yes/no...

Let's ask the following question from curiosity.

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Why does QCD vacuum spontaneously break chiral symmetry?

Does dynamics choose it out of many possibility?
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Interestingly, QCD in chiral limit must be gapless!
('t Hooft, Wess, Stora, Zumino, Frishman, Schwimmer, Banks, Yankielowicz, ..., $\sim$ '80s)

So there is a constraint, though chiral SSB is NOT the unique possibility.

Main result

Now, I want to ask if there are any missing constraints in '80s?

Especially, we usually think of chiral SSB as

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V,$$

but the exotic breaking, (Stern, '97, Kogan, Kovner, Shifman, '98, Kanazawa, '15)

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (\mathbb{Z}_{N_f})_{\mathrm{chiral}},$$

is equally good in view of '80s.

One of the main results is roughly

Theorem

there is an extra constraint, ruling out the (naive) exotic scenario.

Related works: Gaiotto, Kapustin, Komargodski, Seiberg, '17,

Shimizu, Yonekura, '17, ...

Review 't Hooft anomaly matching

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When we have strongly coupled QFT, it's almost hopeless to solve it exactly.

How can we know whether a given theory has mass gap or not?

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This question originally arises in technicololor context.
In one of the lectures by 't Hooft, interesting proposal was given.

We should consider an 't Hooft anomaly
$\equiv$ Global symmetry that cannot be gauged.

Anomaly matching

Let's assume we have a QFT with global symmetry G

Definition ('t Hooft anomaly)

Introduce Background gauge field A for G.

$$Z[A+\delta_\theta A]=\exp\left(\mathrm{i}\mathcal{A}[A,\theta]\right) Z[A].$$

Anomalous phase $\mathcal{A}[A,\theta]$ is nowadays called an 't Hooft anomaly.

Theorem ('t Hooft anomaly matching)

't Hooft anomaly is renormalization-group invariant. *('t Hooft, '79, '80)*
Low-energy effective theory must reproduce the same $\mathcal{A}[A,\theta]$.

QCD and 't Hooft anomaly

Let's see how it works in QCD with massless quarks $m=0$.

Following the idea, we introduce the gauge field A_L for $SU(N_f)_L$:

$$\overline{\psi}\gamma_\nu D_\nu \psi \quad \Rightarrow \quad \overline{\psi}\gamma_\nu \left(D_\nu + \frac{1+\gamma_5}{2} A_L\right)\psi.$$

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$$\overline{\psi}\gamma_\nu D_\nu \psi \quad \Rightarrow \quad \overline{\psi}\gamma_\nu \left(D_\nu + \frac{1+\gamma_5}{2} A_L \right) \psi.$$

Apply the background gauge transformation, $\delta_{\theta_L} A_L = [(\mathrm{d} + A_L), \theta_L]$, then we can find

$$Z[A_L + \delta_{\theta_L} A_L] = Z[A_L] \exp \frac{N_c}{24\pi^2} \int \mathrm{tr} \left[\theta_L \mathrm{d} \left(A_L \mathrm{d} A_L + \frac{1}{2} A_L^3 \right) \right].$$

So, we can compute anomaly exactly without solving QCD!

QCD in chiral limit must be gapless

OK, we computed an 't Hooft anomaly of QCD.
Next question would be, how can we use it?

Anomaly tells "QCD$+A_L$" should be defined on the boundary of 5d manifolds M_5 with $SU(N_f)_L$ bundle:

$$Z_{\mathrm{QCD},\partial M_5}[A_L]\exp\left(N_c\,\text{Chern-Simons}_{5,M_5}[A_L]\right)\text{ is gauge inv.}$$

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Under RG, the CS level N_c cannot be changed as it only takes discrete values.
EFT of QCD must have the same anomaly for gauge invariance.

In ’80’s, anomaly matching is restricted to continuous chiral symmetry.

This turns out to be too restrictive.(X.G. Wen, Kapustin, Thorngren, Cho, Theo, Ryu, ..., ’13∼)
Symmetry G can be any symmetries, either discrete or continuous, higher-form, 2-group, etc.

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Theorem

For 't Hooft anomaly of such discrete/generalized symmetries, low-energy EFT must include (in relativistic setup)

SSB, gapless excitations / CFT, or topological order.

Discrete anomaly of QCD and Stern phase

Orthodox and Exotic chiral SSB

Usually, chiral SSB is caused by the quark bilinear condensate,

$$\overline{\psi}\psi = \overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R.$$

Having VEV, $\langle\overline{\psi}_R\psi_L\rangle \sim \Lambda^3$, this causes SSB,

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V.$$

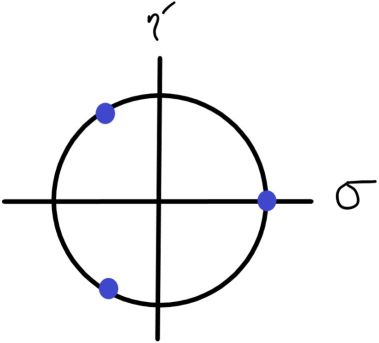
Exotic one (Stern phase) Chiral SSB is

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (\mathbb{Z}_{N_f})_\chi.$$

Continuous part is broken as usual. The point is there is an extra discrete symmetry,

$$\psi_L \to \mathrm{e}^{2\pi\mathrm{i}/N_f}\psi_L,\ \psi_R \to \psi_R.$$

The difference can be seen in σ-η' plane ($N_f = 3$).



This is the usual scenario. There are N_f vacua on the circle with radius Λ^3.

In Stern phase, the vacuum is at the origin and "unique" in this plane.

Order parameter

Since these two have different SSB patterns, we can distinguish them by Landau's order parameter theory.

As we have seen, the usual one is characterized by

$$\langle \bar{\psi}_R \psi_L \rangle \sim \Lambda^3.$$

We can see that $(\mathbb{Z}_{N_f})_\chi$ acts as $\bar{\psi}_R \psi_L \to \mathrm{e}^{2\pi\mathrm{i}/N_f} \bar{\psi}_R \psi_L$, so $(\mathbb{Z}_{N_f})_\chi$ is broken.

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In Stern phase, $\langle \bar{\psi}_R \psi_L \rangle = 0$, but

$$\langle (\bar{\psi}_R \psi_L)(\bar{\psi}_L \psi_R) \rangle \sim \Lambda^6.$$

Within Ginzburg-Landau, both should be possible scenarios.

't Hooft anomaly and Stern phase

Stern phase has the same content of NG bosons because of continuous chiral SSB.
$\Rightarrow$ Anomaly matching in '80's doesn't tell difference.

To emphasize the role of discrete axial symmetry, we look at

$$G^{\mathrm{sub}} = \frac{SU(N_f)_V \times U(1)_V}{(\mathbb{Z}_{N_\mathrm{c}}) \times (\mathbb{Z}_{N_\mathrm{f}})} \times (\mathbb{Z}_{N_\mathrm{f}})_\chi,$$

instead of full chiral symmetry G.

Finding anomaly of G^{sub}, we can rule out exotic scenario!

Contents of background gauge fields

To detect the 't Hooft anomaly of G^{sub},

$$G^{\mathrm{sub}} = \frac{SU(N_f)_V \times U(1)_V}{(\mathbb{Z}_{N_\mathrm{c}}) \times (\mathbb{Z}_{N_\mathrm{f}})} \times (\mathbb{Z}_{N_\mathrm{f}})_\chi,$$

we introduce the background gauge fields (1710.08923, 1711.10487) (ref. Kapustin, Seiberg (2014));

- $SU(N_f)_V$ one-form gauge field: A_f
- $U(1)_V$ one-form gauge field: A_V
- $(\mathbb{Z}_{N_f})_\chi$ one-form symmetry: A_χ
- $(\mathbb{Z}_{N_c})$ two-form gauge field: B_c
- $(\mathbb{Z}_{N_f})$ two-form gauge field: B_f

Not only the usual gauge fields, $A_{V,\chi}$, we need higher-form gauge fields $B_{c,f}$.

Role of two-form gauge fields

Why do we need two-form gauge fields?

Gauging $U(1)_V/\mathbb{Z}_{N_c}$

Gauge group $SU(N_\mathrm{c})$ and the $U(1)_V$ phase rotation of quark have the common elements $\mathbb{Z}_{N_\mathrm{c}}$ since q has the charge $(\Box, 1)$.

$$\begin{aligned} &\text{Gauging } U(1)_B = U(1)_V/\mathbb{Z}_{N_\mathrm{c}} \\ \Leftrightarrow\ & U(1)_V \text{ gauge field} + \text{constraint: possible charges are } (\Box^k, k). \end{aligned}$$

B_c gives the constraint on the possible gauge charges correctly!
The $U(1)_B$ gauge field A_B is given by

$$\mathrm{d}A_B = N_\mathrm{c}(\mathrm{d}A_V + B_\mathrm{c}).$$

Discrete 't Hooft anomaly for massless QCD

After introducing the background gauge fields, the partition function $\mathcal{Z}_{\mathrm{QCD}}$ is no longer gauge invariant.

Gauge invariance requires to add 5d SPT phase:

$$\mathcal{Z}_{\mathrm{QCD}} \exp\left(\frac{N_{\mathrm{f}}}{(2\pi)^2}\int A_\chi \wedge \mathrm{d}A_B \wedge B_{\mathrm{f}}\right).$$

This says that the baryon number conservation is anomalously violated under $(\mathbb{Z}_{N_{\mathrm{f}}})_L$ and $SU(N_{\mathrm{f}})/\mathbb{Z}_{N_{\mathrm{f}}}$ gauge field:

$$\partial_\mu J^\mu_B = \frac{N_{\mathrm{f}}}{(2\pi)^2}\mathrm{d}A_\chi \wedge B_{\mathrm{f}}.$$

This is the discrete version of baryon number violation by electroweak instantons.

Skyrmions in ordinary chiral SSB

Let us see how the discrete anomaly is matched for ordinary case:

$$G=\frac{SU(N_\mathrm{f})_L\times SU(N_\mathrm{f})_R\times U(1)_V}{\mathbb{Z}_{N_\mathrm{c}}\times\mathbb{Z}_{N_\mathrm{f}}}\to H=\frac{SU(N_\mathrm{f})_V\times U(1)_V}{\mathbb{Z}_{N_\mathrm{c}}\times\mathbb{Z}_{N_\mathrm{f}}}$$

The nonlinear Langrangian has the target space $G/H=SU(N_\mathrm{f})$.

Since $\pi_3(G/H)=\mathbb{Z}$, there are Skyrmions with the skyrmion current

$$J_\mathrm{skyrmion}=\frac{1}{24\pi^2}\mathrm{tr}[(U^\dagger \mathrm{d}U)^3].$$

$\Rightarrow$ $\mathrm{d}J_\mathrm{skyrmion}[A_\chi,B_\mathrm{f}]=\mathrm{d}J_B[A_\chi,B_\mathrm{f}]$, and the anomaly matching is satisfied. (1807.07666[hep-th])

This is an extension of the $U(1)_V$-$SU(N_\mathrm{f})_L$-$SU(N_\mathrm{f})_L$ anomaly matching in the chiral Lagrangian. (ref. Witten, 1983)

Exotic chiral symmetry breaking

We now consider the Stern phase, where

$$G \to G^{\mathrm{sub}} = \frac{SU(N_F)_V \times U(1)_V}{(\mathbb{Z}_{N_\mathrm{c}}) \times (\mathbb{Z}_{N_\mathrm{f}})} \times (\mathbb{Z}_{N_\mathrm{f}})_L.$$

The target space of the nonlinear Lagrangian is

$$G/G^{\mathrm{sub}} = SU(N_\mathrm{f})/\mathbb{Z}_{N_\mathrm{f}}.$$

$\Rightarrow$ We can obtain the effective theory by gauging $\mathbb{Z}_{N_\mathrm{f}}$ in the ordinary chiral Lagrangian.

U : Nonlinear sigma field $\in SU(N_\mathrm{f}), a_\chi$: $\mathbb{Z}_{N_\mathrm{f}}$ dynamical gauge field

Mismatch of anomaly in Stern phase

Nontrivial homotopy: $\pi_3(G/G^{\mathrm{sub}}) = \mathbb{Z}$, $\pi_1(G/G^{\mathrm{sub}}) = \mathbb{Z}_{N_\mathrm{f}}$. There are skyrmions with the current J_{skyrmion}. There are also $\mathbb{Z}_{N_\mathrm{f}}$ vortices.

Under the background gauge fields, the anomalous violation of J_{skyrmion} is

$$\mathrm{d}J_{\mathrm{skyrmion}} = \frac{N_\mathrm{f}}{(2\pi)^2}\mathrm{d}a_\chi \wedge B_\mathrm{f} \neq \mathrm{d}J_B.$$

Therefore, the anomaly matching is not satisfied.

$\Rightarrow$ We rule out the Stern phase from the possible QCD vacua even at finite densities. (1807.07666[hep-th])

Bonus 1: QCD + Yukawa

Let's play another game. Add a complex scalar to QCD with Yukawa coupling,

$$\phi(\overline{\psi}_R\psi_L)+\phi^*(\overline{\psi}_L\psi_R).$$

Combined with the $U(1)$ rotation of ϕ, we find that this theory enjoys the Stern-phase symmetry $G^{\rm sub}$.

How many vacua do you expect in this theory?

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How many vacua do you expect in this theory?

Of course, it depends on the Lagrangian of scalar sector, but this theory has the discrete anomaly. So, it should either be

- N_f vacua by $(\mathbb{Z}_{N_f})_\chi \to 1$.
- gapless,
- topological.

Bonus 2: $SU(N)$ ver. Haldane conjecture

Consider the $SU(N)$ spin chain with p-box rep. on each site.

Nonlinear sigma model

Under certain assumptions, Bykov ('12,'13) and Lajko et al. ('17) showed that the LEFT is $SU(N)/U(1)^{N-1}$ NLσM:

$$S = \frac{1}{g^2}\sum_{i=1}^{N}\int |(\mathrm{d}+\mathrm{i}a_i)z_i|^2 + \mathrm{i}\sum_{i=1}^{N}\frac{\theta_i}{2\pi}\int \mathrm{d}a_i + \cdots .$$

Here, $U=[z_1,\ldots,z_N]$ is an $SU(N)$ matrix, and

$$\theta_k = \frac{2\pi p}{N}k.$$

$SU(N)$ Wess-Zumino-Witten model

Consider the level-p $SU(N)$ WZW model,

$$S = \frac{k}{8\pi^2}\int_{M_2} |\mathrm{d}\mathcal{U}|^2 + \frac{\mathrm{i}p}{12\pi}\int_{M_3} \mathrm{tr}[(\mathcal{U}^{-1}\mathrm{d}\mathcal{U})^3].$$

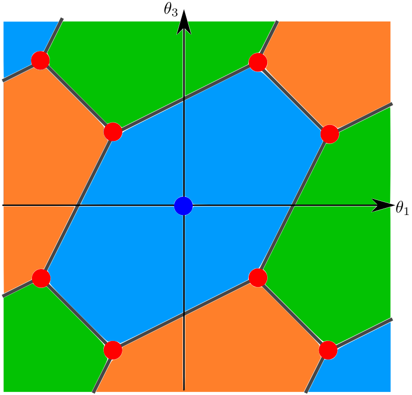
The model has continuous chiral symmetry, but we pay attention to its subgroup:

$$\frac{SU(N)_L \times SU(N)_R}{\mathbb{Z}_N} \supset PSU(N)_V \times (\mathbb{Z}_N)_\chi.$$

Double-trace potential, $|\mathrm{tr}(\mathcal{U})|^2$, only respects this subgroup.
This deformation continuously interpolates between WZW and $SU(N)/U(1)^{N-1}$ sigma model! (YT, Sulejmanpasic, 1805.11423)

WZW term is same with Skyrmion current, so we have $\mathbb{Z}_N$ anomaly.

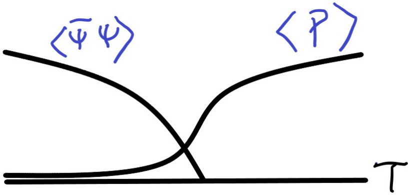
Phase diagram of $SU(3)/U(1)^2$ NLσM



- Different colors = different SPT phases (by global inconsistency)
- Red blobs = $SU(3)_1$ WZW model, or trimerized phase (by anomaly matching) (YT, Sulejmanpasic, 1805.11423)

Bonus 3: Phase diagram of $\mathbb{Z}_N$-QCD

In QCD, fundamental quarks break center, so we don't have an order parameter for center symmetry in strict sense.



But, it's close! We are tempted to say deconfinement and chiral restoration occurs almost at the same temperature!

Center symmetry and fundamental quarks

OK, give up a usual temperature, but we can cook up a nice setup, $\mathbb{Z}_N$-QCD. (Kouno et al. 2012, Poppitz, Sulejmanpasic, '13, Iritani, Misumi, Itou, '15)

Prepare $N_c = N_f = N$ quarks, and put the boundary condition on the quark field as $(f = 1, \dots, N)$

$$q_f(\boldsymbol{x}, x_4 + L) = \omega^f q_f(\boldsymbol{x}, x_4).$$

This theory has the $(\mathbb{Z}_N)_{\mathrm{shift,center}}$ symmetry, defined by

$$\Phi = \mathrm{tr_c}\left[P \exp\left(\mathrm{i}\int_{S^1} a\right)\right] \mapsto \omega\, \Phi,\ q_f \mapsto q_{f+1}.$$

$\mathbb{Z}_N$-QCD

Using operators of QCD, the partition function of $\mathbb{Z}_N$-QCD is

$$\mathcal{Z}_{\mathbb{Z}_N-\mathrm{QCD}} = \mathrm{tr}_{\mathcal{H}}\left[\mathrm{e}^{-L(\widehat{H}-\mu\widehat{Q})}\exp\left(\mathrm{i}\sum_{f=1}^{N}\frac{2\pi f}{N}\widehat{Q}_f\right)\right].$$

In this setup, discrete anomaly of QCD constrains the possible phases of $\mathcal{Z}_{\mathbb{Z}_N-\mathrm{QCD}}$:

(Discrete) chiral symmetry is broken in the center-symmetric phase, e.g.

$$T_{\mathrm{deconf}} \le T_{\mathrm{chiral}}$$

for massless $\mathbb{Z}_N$-QCD. (1710.08923, 1711.10487)

(Similar, related results for pure YM, Bifundamental QCD, adjoint QCD, etc.: Gaiotto, Kapustin, Komargodski, Seiberg (1703); Tanizaki, Kikuchi (1705); Komargodski, Sulejmanpasic, Unsal (1706); Shimizu, Yonekura (1706); ...)

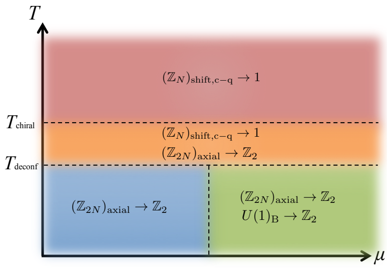
Comment on Possible phase diagrams

We have a mixed 't Hooft anomaly between $(\mathbb{Z}_N)_{\rm center}$, $\frac{U(1)_F^{N-1}\times U(1)_V}{(\mathbb{Z}_N)_C\times(\mathbb{Z}_N)_F}$, and $(\mathbb{Z}_{2N})_{\rm axial}$.

1. System is conformal or breaks some of symmetries
2. When anomaly is matched by SSB,

$$T_{\rm deconf}(\mu)\le T_{\rm chiral}(\mu)$$

if flavor is unbroken.



Take Home Message

Anomaly, Anomaly, Anomaly!